Name _____

Asymptotes of Rational Functions

Vertical Asymptotes

Definition: The line $x = a$ is a vertical asymptote of the graph of $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as x approaches "a" from either the left or the right.

→ Happen at the value(s) for which the domain is undefined.
- → Set the denominator=0 and solve for x.
- → (Note that these x-values are only vertical asymptotes if, when x is plugged back in, the numerator $\neq 0$)

→ The graph of a rational function will NEVER cross a vertical asymptote.

EXAMPLES:
Find the vertical asymptotes for the following functions:

1. $f(x) = \dfrac{1}{x+2}$

2. $f(x) = \dfrac{x-1}{(x+2)(x+1)}$

3. $f(x) = \dfrac{x}{x^2+1}$

4. $f(x) = \dfrac{2x^2-3x-1}{x-2}$

5. $f(x) = \dfrac{2x^2+7x-4}{x^2+x-2}$

6. $f(x) = \dfrac{5x}{x^2-4}$

7.

x	f(x)
-2	1
-2.5	2.222
-2.9	11.837
-2.99	119.84
-2.999	1199.8

As x approaches _____
from the _____,
f(x) approaches _____.

x	f(x)
-4	-1.333
-3.5	-2.545
-3.1	-12.16
-3.01	-120.2
-3.001	-1200

As x approaches _____
from the _____,
f(x) approaches _____.

Horizontal Asymptotes

Definition: The line $y = b$ is a horizontal asymptote of the graph of $f(x)$ if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.

→ If the degree of the numerator is less than the degree of the denominator (in other words, the function is **proper**), the horizontal asymptote is $y = 0$

→ If the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is the ratio of the leading coefficients.

→ If the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

→ Horizontal asymptotes describe the <u>end</u> behavior of a function. A graph MAY (or might not) cross a horizontal asymptote between the ends, but the graph must level off at one or both ends.

EXAMPLES:

Find the horizontal asymptotes for the following functions:

1. $f(x) = \dfrac{4}{x^2+1}$

2. $f(x) = \dfrac{2x}{3x^2+1}$

3. $f(x) = \dfrac{2x+1}{x+1}$

4. $f(x) = \dfrac{5x^2+1}{2x^2-8}$

5. $f(x) = \dfrac{3x^3-5x^2+4x-5}{3x+1}$

6. $f(x) = \dfrac{x^2-9}{x+2}$

7.

x	f(x)
-1	1
-10	-0.125
-100	-0.0102
-1000	-0.001

$y \to$ _____ as $x \to$ _____

x	f(x)
1	0.333
10	0.0833
100	0.0098
1000	0.0009

$y \to$ _____ as $x \to$ _____

Oblique/Slant Asymptotes

The graph of a rational function has a slant asymptote if the degree of the numerator is exactly one more than the degree of the denominator.

→ Use polynomial long division to find the slant asymptotes. (Ignore the remainder).

→ You cannot have BOTH a slant asymptote and a horizontal asymptote.

EXAMPLES:

Find the slant/oblique asymptotes for the following functions:

1. $f(x)=\dfrac{3x^4-x^2}{x^3-x^2+1}$

2. $f(x)=\dfrac{x^2-x-2}{x-1}$

3. $f(x)=\dfrac{x^2-2}{x-1}$

4. $f(x)=\dfrac{8x^3-x^2+3x+2}{4x^2-1}$

MIXED PRACTICE:

Find ALL asymptotes (vertical, horizontal, and/or slant) of the given functions.

1. $f(x)=\dfrac{3x-2}{(x-1)(x+4)}$

2. $f(x)=\dfrac{x^2-16}{x^2+4x-21}$

Find the vertical, horizontal, and oblique asymptotes (if any).

11. $R(x) = \dfrac{4x}{x-3}$

12. $R(x) = \dfrac{5x^2}{3+x}$

13. $H(x) = \dfrac{-4x^2}{(x-2)(x+4)}$

14. $G(x) = \dfrac{6}{(x+3)(4-x)}$

15. $F(x) = \dfrac{3x(x-1)}{2x^2-5x-3}$

16. $Q(x) = \dfrac{-x(1-x)}{3x^2+5x-2}$

17. $R(x) = \dfrac{x}{x^3-8}$

18. $R(x) = \dfrac{x}{x^4-1}$

19. $H(x) = \dfrac{3x^2+x}{x^2+4}$

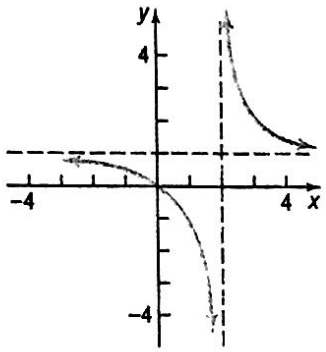
20. $G(x) = \dfrac{x-3}{x^4+1}$

21. $R(x) = \dfrac{3(x^2-x-6)}{4(x^2-9)}$

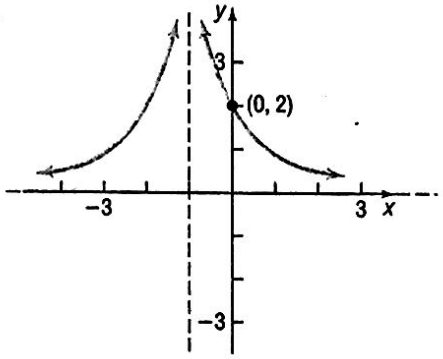
22. $F(x) = \dfrac{-2(x^2-4)}{3(x^2+4x+4)}$

In Problems 23–28, use the graph shown to find:
(a) The domain and range of each function *(b) The intercepts, if any* *(c) Horizontal asymptotes, if any*
(d) Vertical asymptotes, if any *(e) Oblique asymptotes, if any*

23.



24.



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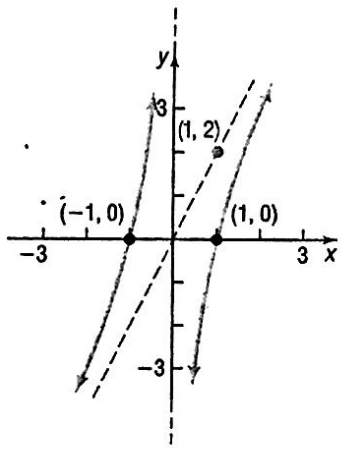

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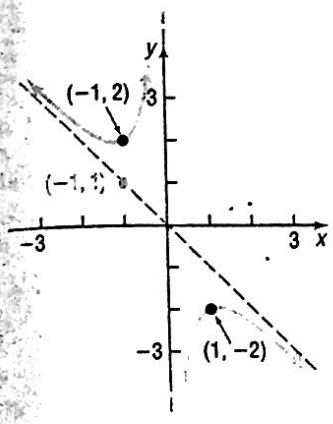

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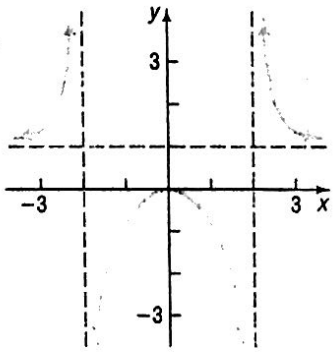

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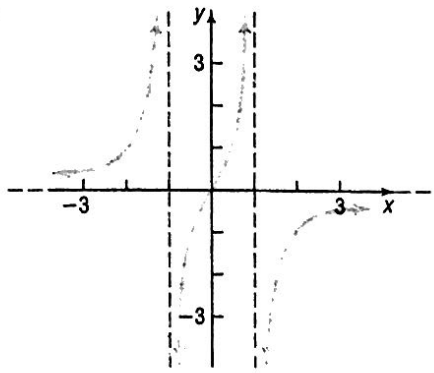

In Problems 41–52, find the vertical, horizontal, and oblique asymptotes, if any, of each rational function.

41. $R(x) = \dfrac{3x}{x+4}$

42. $R(x) = \dfrac{3x+5}{x-6}$

43. $H(x) = \dfrac{x^3-8}{x^2-5x+6}$

44. $G(x) = \dfrac{x^3+1}{x^2-5x+6}$

45. $T(x) = \dfrac{x^3}{x^4-1}$

46. $P(x) = \dfrac{4x^2}{x^3-1}$

47. $Q(x) = \dfrac{2x^2-5x-12}{3x^2-11x-4}$

48. $F(x) = \dfrac{x^2+6x+5}{2x^2+7x+5}$

49. $R(x) = \dfrac{6x^2+7x-5}{3x+5}$

50. $R(x) = \dfrac{8x^2+26x-7}{4x-1}$

51. $G(x) = \dfrac{x^4-1}{x^2-x}$

52. $F(x) = \dfrac{x^4-16}{x^2-2x}$